C	Pierce's gain parameter, $(I_0K/4V_0)^{1/3}$.
I_0	Average electron convection current.
V_0	Average beam voltage.
δ	Normalized propagation constant of Pierce.
b	$(u_0-v_p)/Cu_0$, where u_0 is the average electron velocity, $\sqrt{2\eta V_0}$.
QC	Space charge parameter of Pierce.
V_c	Circuit voltage of Pierce (rms).
V	Total voltage of Pierce (rms).
i	Alternating electron convection current, positive if electrons flow toward positive z.
v	Alternating beam velocity, positive toward positive z.
N	l/λ, circuit length in wavelengths.
η	Ratio of electronic charge, e, to electronic mass, m, 1.76×10^{11} coulomb/kg.
L	Total circuit loss, 54.6 CNd decibels.
N	Number of circuit wavelengths.
T_c	Cathode temperature in degrees Kelvin.
e	Electronic charge, coulomb.
j	Symbolizes an imaginary number $\sqrt{-1}$.
ω_q	Reduced plasma frequency, $p\omega_p$ or $p\sqrt{\eta I_0/\epsilon_0 u_0\sigma}$, where σ = beam cross-sectional area, p = plasma reduction factor.
β_q	ω_q/u_0.
$\xi(\delta)$, $\tau(\delta, QC)$, $\phi(\delta, QC)$, $D(\delta, QC)$	parameters defined for convenience after (8).

The Nature of the Uncorrelated Component of Induced Grid Noise*

T. E. TALPEY†, MEMBER, IRE, AND A. B. MACNEE‡, MEMBER, IRE

Summary—**An investigation of induced grid noise in vacuum tubes has been made. It was found that the uncorrelated component of grid noise can be explained in terms of electrons elastically reflected from the plate of the tube. Experimental and theoretical justifications of this explanation are presented. The accuracy of methods for predicting grid noise from measurements of input admittance is affected because of a component of input admittance arising from reflected electrons. A table is included showing typical (measured) values of the induced grid noise of eleven modern receiving tubes.**

Introduction

Two types of vacuum tube noise, shot noise and induced grid noise, are of importance in the design of low-noise, high-frequency amplifiers. Shot noise is the fluctuating component of plate current caused by random variations in the cathode emission rate. Induced grid noise is generated by fluctuations in the number of current pulses induced in the grid circuit by the passage of electrons between grid wires. At the higher operating frequencies, induced grid noise is the limiting factor in low-noise amplifier design.[1,2]

For many years the theory of induced grid noise has been in a rather unsatisfactory state. Calculations based directly on electron transit times yield induced grid noise magnitudes which are consistently low, often by a factor of two or three, (see Fig. 1), while predictions based on measurements of the input admittance are

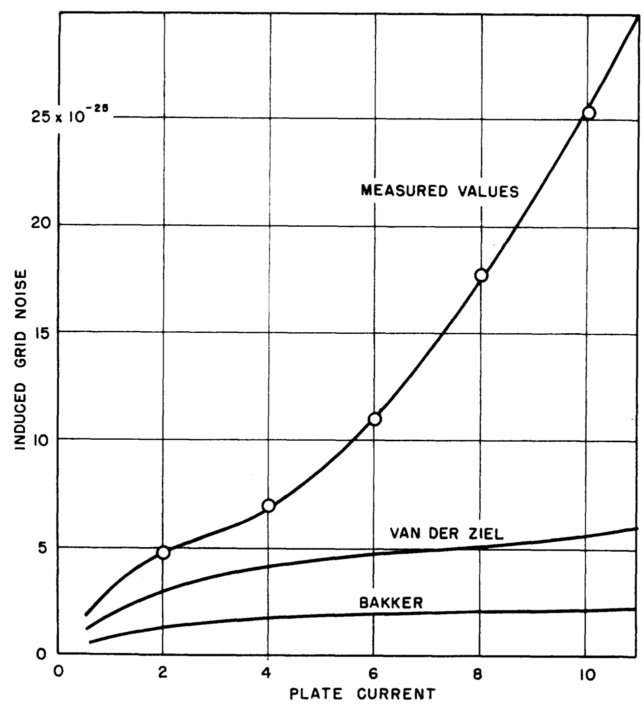

Fig. 1—Comparison between measured and calculated values of mean square induced grid noise current as a function of plate current for one section of a 6J6 double triode at 30 mc. Grid noise is expressed in amperes squared per unit bandwidth, and the plate current in ma.

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[1] Frequencies above about 15mc for modern miniature tubes.

[2] H. Wallman, A. B. Macnee, and C. P. Gadsden, "A low-noise amplifier," Proc. I.R.E., vol. 36, pp. 700–708; June, 1948.

found to provide a fair amount of agreement with measured grid noise values.[3–5]

It is evident from experimental studies that a large component of the induced grid noise is uncorrelated with the shot noise in the plate current stream.[6,7] Previous writers have suggested two explanations for the origin of this uncorrelated component:

1. Total emission noise[8]—fluctuating currents induced in the grid circuit by electrons returned to the cathode before reaching the potential minimum.

2. Induced partition noise[9]—fluctuations arising because of electron-trajectory variations and inhomogeneities in the electrode structure. It can be thought of as arising from fluctuations in average transit angle.

Consideration of the position of the potential minimum relative to electrode spacings indicates that total emission noise is negligible with respect to the total induced grid noise in tubes such as the 6AK5. Bell[9] has estimated the magnitude of noise to be expected from electron-trajectory variations and concludes that the effect is small under normal operating conditions. The validity of this conclusion has not been completely substantiated by experiment, but it appears that these two explanations alone are not adequate to account for the *observed* excess grid noise.

This paper presents the results of theoretical and experimental studies which indicate that a major portion of the uncorrelated component of induced grid noise is caused by fluctuations in a small number of electrons which are elastically reflected by the plate. These electrons are reflected with sufficient energy to enable them to return through the grid, inducing additional current pulses in the grid circuit. This current increases the input admittance and the induced grid noise. A brief discussion concerning the accuracy of grid noise predictions based on measurements of input conductance or susceptance is included.

For the benefit of the design engineer a table has been included showing typical measured magnitudes of induced grid noise and plate noise for a number of modern miniature receiving tubes.

Analysis of Induced Grid Noise

Theoretical studies, published first by Bakker[3] and recently extended by van der Ziel,[10] show that fluctuations in the cathode emission of a planar triode should produce a mean square induced grid-noise current

$$\overline{i_g^2} = \overline{i_k^2}\left(\frac{\omega\tau_1}{3}\right)^2\left[1 + 2\left(\frac{\tau_2}{\tau_1}\right)\right]^2. \qquad (1)$$

In this equation

$\overline{i_k^2}$ = mean square space-charge-reduced shot-noise component of cathode current, I_k
$= 2eI_k\Gamma^2\Delta f$
τ_1 = transit time from potential minimum to grid-plane
τ_2 = transit time from grid-plane to plate
Γ^2 = space-charge reduction factor
e = charge on an electron $= 1.6\times10^{-19}$ coulomb
Δf = bandwidth in cps.

Eq. (1) is plotted for a typical case in Fig. 1. By a method similar to that described by Goldman[11] it has been shown that to a first approximation the induced grid noise can be expressed as[12]

$$\overline{i_g^2} = \frac{2I_b\Delta f\Gamma^2}{e}\cdot\left|S(\omega)\right|^2, \qquad (2)$$

where

$$S(\omega) = 2\pi G(\omega) = \int_{-\infty}^{+\infty} F(t)e^{-j\omega t}dt. \qquad (3)$$

$G(\omega)$ is the Fourier transform of $F(t)$, the current pulse induced in the grid by the passage of a single electron from cathode to plate [see Fig. 2(a)]. Comparison of (1) and (2) reveals the form of $|S(\omega)|^2$. Since the area of the grid current pulses must be zero, their power spectra and the mean square induced grid-noise current are both proportional to the square of frequency at small transit angles. This observation has been verified experimentally numerous times.[3–6]

The lack of agreement between measured and calculated values of induced grid noise as exemplified by Fig. 1 can be explained in terms of electrons reflected by the plate. It has long been known[13,14] that electrons which are elastically reflected at the plate *of a diode* cause an increase in the shot noise. Strangely enough, no study has been published showing the effect of reflected electrons on grid noise.

An electron which is elastically reflected from the plate has sufficient energy to penetrate the retarding field that it meets between grid and plate. It will very likely succeed in passing back between the grid wires

[3] C. J. Bakker, "Fluctuations and electron inertia," *Physica*, vol. 8, pp. 23–43; January, 1941.
[4] D. O. North and W. R. Ferris, "Fluctuations induced in vacuum tube grids at high frequencies," Proc. I.R.E., vol. 29, pp. 49–50; February, 1941.
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[6] R. Q. Twiss and Y. Beers, "Minimal Noise Circuits," *Vacuum Tube Amplifiers*, vol. 18, Ch. 13, M.I.T. Rad. Lab. Series, McGraw-Hill Book Company, New York N.Y., 1948.
[7] A. van der Ziel, "Noise suppression in triode amplifiers," *Canad. Jour. Tech.*, vol. 29, pp. 540–553; December, 1951.
[8] A. van der Ziel and A. Versnel, "Induced grid noise and total emission noise," *Philips Res. Rep'ts*, vol. 3, pp. 13–23; February, 1948.
[9] R. L. Bell, "Negative grid partition noise," *Wireless Eng.*, vol. 25, pp. 294–297; September, 1948.
[10] A. van der Ziel, "Induced grid noise in triodes," *Wireless Eng.*, vol. 28, pp. 226–227; July, 1951.
[11] S. Goldman, "Frequency Analysis, Modulation and Noise," McGraw-Hill Book Company, New York, N.Y., 356 ff; 1948.
[12] T. E. Talpey, "A Study of Induced Grid Noise," Doctoral Thesis, July 1953, available on microfilm from University Microfilms, Ann Arbor, Michigan.
[13] D. O. North, "Fluctuations in space-charge-limited currents at moderately high frequencies, Part II, diodes and negative grid triodes," *RCA Rev.*, vol. 4, pp. 441–472; April, 1940; vol. 5, pp. 106–124; July, 1940.
[14] G. E. Duvall, "The Effects of Transit Angle on Shot Noise in Vacuum Tubes," M.I.T. Res. Lab. of Electronics, Tech. Rep. No. 82, Sept. 8, 1948.

before it loses its cathode-directed energy and is finally drawn back to the plate again. The pulse of current induced in the grid circuit by such a reflected electron will be approximately three times as long as the pulse produced by an ordinary electron, as indicated in Fig. 2(b).

By the application of two theorems from the study of Fourier integrals[15] it is easily shown that the power spectrum of the complex pulse shown in Fig. 2(b) is given by the following expression:

$$|S_r(\omega)|^2 = |S(\omega) - S(-\omega)e^{j2\omega(\tau_1+\tau_2)} + S(\omega)e^{-j2\omega(\tau_1+\tau_2)}|^2 \quad (4)$$

where $|S(\omega)|^2$ is the power spectrum of the first third of the pulse, up to the point where the electron first arrives at the plate.

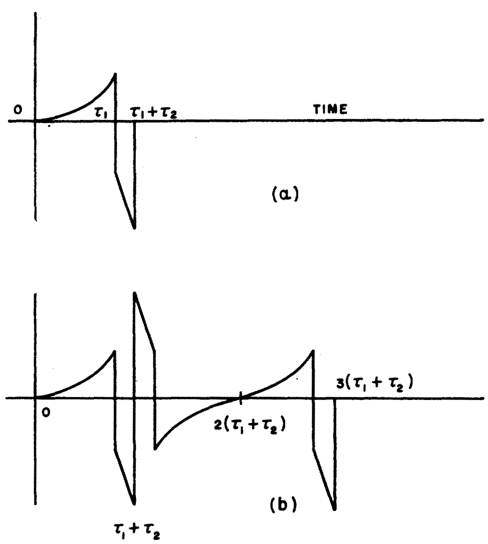

Fig. 2—(a) Current pulse induced in the grid circuit of an ideal space-charge-limited triode by the passage of a single electron, (b) Current pulse induced in the grid circuit by an electron which traverses the tube, is elastically reflected at the plate, succeeds in getting back close to the potential minimum, and then returns to the plate.

Now $S(\omega)$ can be expanded in a power series in terms of $\omega\tau_1$ as follows:

$$S(\omega) = a_1\omega\tau_1 + a_2(\omega\tau)^2 + a_3(\omega\tau_1)^3 + \cdots \quad (5)$$

where the factors a_1, a_2, etc. involve terms in (τ_2/τ_1), $(\tau_2/\tau_1)^2$, etc., as in (1). Assuming that the transit angle $\omega\tau_1$ is considerably smaller than one radian,[16] we can neglect all terms except the first in (5) and write

$$S(\omega) \cong -S(-\omega) \cong a_1\omega\tau_1. \quad (6)$$

Eq. (4) then becomes

$$|S_r(\omega)|^2 \cong |S(\omega)|^2 |1 + 2\cos 2\omega(\tau_1 + \tau_2)|^2. \quad (7)$$

If $2\omega(\tau_1+\tau_2)$ is also considerably less than one radian, the cosine is approximately unity and we obtain the relationship

[15] See, for example, E. A. Guillemin, "The Mathematics of Circuit Analysis," John Wiley & Sons, New York, N.Y., Ch. VII, Article 22; 1949.

[16] At 30 mc, the transit angles of all the miniature tubes studied are well below 0.1 radian.

$$|S_r(\omega)|^2 \cong 9|S(\omega)|^2. \quad (8)$$

The reflected electrons thus produce an induced grid noise component given approximately by

$$\overline{i_g^2} = \frac{2\Delta f}{e}(rI_b)(9)|S(\omega)|^2, \quad (9)$$

where r is the reflection coefficient of the plate, that is, the fraction of incident electrons which are elastically reflected.

Since fluctuations in the number of reflected electrons are independent of fluctuations in cathode emission, the induced grid noise components given by (2) and (9) add quadratically, giving as an approximate expression for the total induced grid noise at small transit angles

$$\overline{i_g^2} = \frac{2I_b\Delta f}{e}|S(\omega)|^2[\Gamma^2 + 9r]. \quad (10)$$

Logarithmic extrapolation of experimental data reported by Farnsworth[17] indicate that $r = 0.03$ is a reasonable estimate of the reflection coefficient for plate voltages of 100 to 150 volts. Nominal values of Γ^2 lie near 0.1, so that the bracket in (10) becomes

$$[\Gamma^2 + 9r] = [0.1 + .27] = 0.37.$$

The reflected electrons in this case have caused an approximately four-fold increase in induced grid noise. We thus conclude that the reflected electrons are entirely capable of producing the observed excess of measured grid noise over values predicted by earlier theories.

Experimental Verification

The effect of reflected electrons on induced grid noise was verified experimentally by measuring the induced grid noise of a type 6AS6 pentode as a function of suppressor voltage; the results of these measurements are shown in Fig. 3, on the following page.

The induced grid noise increases by a factor of six or so as the suppressor voltage varies from +20 to −20 volts. When the suppressor is negative, it creates a retarding field and some electrons are reflected before they reach the plate. As the suppressor is made more negative, more electrons are reflected until at about −10 volts they are all reflected and the plate current drops to zero. Those electrons which are not captured by the screen travel on toward the grid and induce additional current pulses in the grid circuit.

The correlation between induced grid noise and reflected electrons is even more striking if the data of Fig. 3 are plotted in a different manner. The deficiency in plate current with respect to its asymptotic value at positive suppressor voltages is a measure of the number of electrons which are artificially reflected by the field of the suppressor before they can reach the plate. The grid

[17] H. E. Farnsworth, "Energy distribution of secondary electrons from copper, iron, nickel and silver," *Phys. Rev.*, vol. 31, pp. 405–422; March, 1928

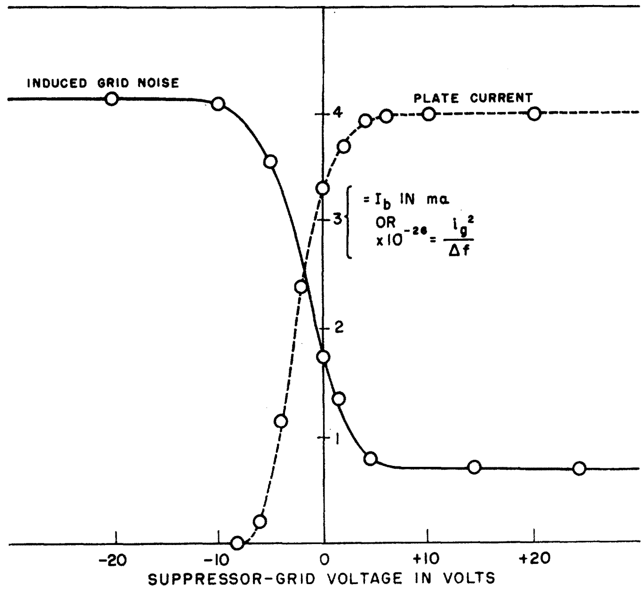

Fig. 3—Mean square induced grid-noise current vs suppressor-grid voltage for a 6AS6 at 30 mc with a fixed control grid voltage and a constant cathode current. The variation in *plate* current is also shown (dashed curve).

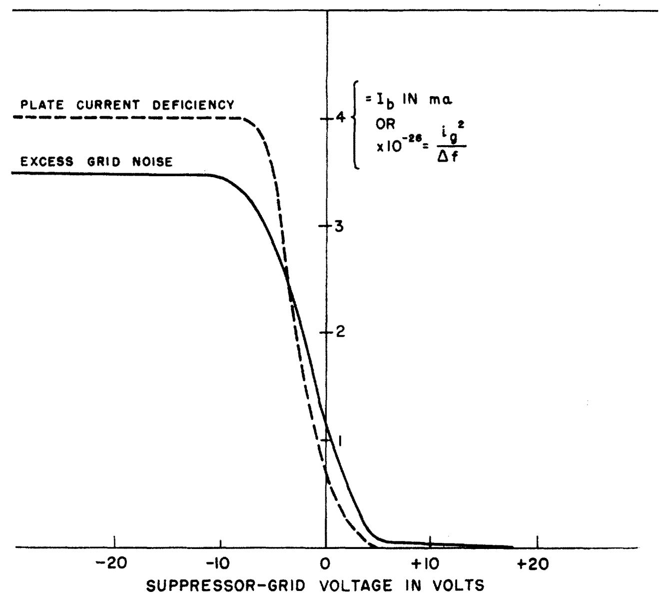

Fig. 4—Excess grid noise and deficiency in plate current (dashed curve) vs suppressor-grid voltage.

noise in excess of its asymptotic value at positive suppressor voltages should be directly proportional to the number of artificially reflected electrons according to (10). When the deficiency of plate current and the excess grid noise are both plotted as a function of suppressor voltage on the same set of coordinates the correlation is excellent, as Fig. 4 clearly shows.

It is significant that the grid noise is appreciably larger for zero suppressor voltage than for positive voltages. When the suppressor is at the same potential as the cathode, the resulting electric field is able to deflect a few electrons sufficiently to prevent them from reaching the plate.[18] These electrons are returned to the vicinity of the control grid and thus cause an increase in the grid noise. If the suppressor connection is brought out to a separate pin, it should be connected to the plate and screen. This eliminates the possibility of reflected electrons being produced by deflection in the screen-suppressor region yet preserves the obstacles presented by the suppressor and screen wires to the return of reflected electrons from the plate. This connection was tried in a 30-mc cascode amplifier[2] employing a 6AS6 input stage. It was found that the noise factor could be reduced from 2.95 to 2.25 db by changing the suppressor connection from the cathode to the plate.

Reflected Electrons and Input Admittance

There is a direct relationship between the electrons which are elastically reflected at the plate of a vacuum tube and the transit-time component of input admittance. This can be demonstrated qualitatively in the following manner. A signal applied to the grid of a tube causes variations in the control-grid voltage which are accompanied by variations in the plate current. For small signals it can be assumed that the reflection coefficient of the plate is constant, so that variations in the plate current will produce proportional variations in the number of reflected electrons.[19] It follows that these reflected electrons must produce, in the grid circuit, a varying induced current which is proportional to the grid voltage. The component of this induced current which is in phase with the grid voltage produces additional input conductance; while the quadrature component produces additional input susceptance. When a measurement is made of the input admittance of a tube, the value obtained includes a component due to reflected electrons as well as the component due to the main electron stream.[20]

Two methods have been advanced for predicting grid noise from measurements of the input admittance. One of these methods[3,4] expresses the grid noise in terms of the transit-time component, G_g, of input conductance:

$$\overline{i_g^2} \cong 4kT\beta G_g \Delta f. \tag{11}$$

The value of the quantity β is usually taken as 5.0, although it is a function of the cathode temperature and certain geometrical factors. Unfortunately, because of lead inductance effects it is frequently more difficult to measure G_g than it is to measure the grid noise directly.

[18] The deflection and subsequent reflection are caused by the combined field of the screen and suppressor grids. The reflection takes place just in front of the suppressor grid. The mechanism is similar to that described by W. G. Dow, "Fundamentals of Engineering Electronics," 2nd Ed., John Wiley & Sons, New York, N. Y., pp. 28–29; 1952.

[19] Sizable variations are being considered here; they should not be confused with the minute random fluctuations which give rise to induced noise. The random fluctuations are superimposed on the variations in plate current.

[20] There is also a component due to feedback from the cathode lead inductance. This feedback usually produces considerable input loading but negligible additional noise.

A second method of predicting grid noise makes use of the following expression:[5]

$$\overline{i_g^2} \cong \overline{i_p^2}\left(\frac{\omega C_e}{g_m}\right)^2, \tag{12}$$

where C_e is the space-charge component of input capacity—*i.e.*, the difference between the input capacity when the tube is operating under normal conditions and the input capacity with the tube biased beyond "cut-off."

Both (11) and (12) imply complete correlation between grid noise and plate noise. Both of these equations were derived under the assumption that both grid noise and plate noise owe their origin to fluctuations in the primary electron stream passing the grid. These fluctuations are affected by space charge in the cathode-grid region and are often said to be "space-charge reduced." On the other hand, fluctuations in the number of reflected electrons are not influenced by space charge in the input region. We are thus led to the conclusion that the grid noise produced by reflected electrons, while linearly related to a component of input admittance, must be related in a slightly different manner than indicated by (11) or (12). The use of these equations for predicting the total induced grid noise consequently involves a certain amount of inaccuracy. Further study of the connection between input admittance and reflected electrons is necessary before the magnitude of the error can be ascertained.

The Induced Grid Noise of Miniature Receiving Tubes

During the course of the research leading to the formulation of the above theory many measurements were made of the induced grid noise of a variety of commercially available receiving tubes. Table I presents a summary of these measurements. Because of its usefulness in network calculations, the induced grid noise values are expressed in terms of an equivalent grid noise conductance (βG_g), based on the representation defined by (11). (No claim for the measurement of G_g alone is intended.) Measurements were also made of the plate noise of these tubes, and the results are presented in Table I in terms of an equivalent shot noise resistance.[13]

The method employed for the measurement of induced grid noise was essentially the same as that described by Bakker.[3] A resonant capacitor was connected from plate to cathode of the tube under test to short-circuit the plate noise and prevent feedback, and the grid was connected directly to the input of a high-gain low-noise amplifier. Noise currents induced in the grid circuit of the tube under test were compared with noise from a temperature-limited diode. The change which occurred in the impedance level of the input circuit when the tube under test was turned on (loading due to transit-time and lead inductance effects) required that a correction be applied to the noise diode reading. The correction was determined from the effect of this loading on the amplifier output level. At 30 mc the effect of lead inductance on *grid noise* is negligible.

The values given in Table I represent the averages of measurements taken on a few tubes of each type, with particular values of voltage and current. Values for any given tube may differ considerably (20 to 30 per cent) from these data, even under the same operating conditions.

The use of a lower grid bias to obtain more plate current was found to cause an increase in grid noise in approximately the same ratio as the g_m was increased. Raising the plate voltage (leaving bias fixed) increases g_m without any appreciable rise in grid noise. It follows that for a given plate-current value, a high plate voltage and high negative grid bias are desirable for the attainment of a low noise figure.

Summary

Experimental and theoretical evidence indicate that the origin of a major portion of the uncorrelated component of induced grid noise is that small fraction of the electron stream which is elastically reflected at the plate of a vacuum tube. It has been shown qualitatively that

TABLE I

Measured Values of Induced Grid Noise at 30mc

Tube type	Number examined	Induced grid noise: Equivalent noise conductance βG_g in micromhos			Plate current I_b in ma	Equivalent shot noise resistance Req.	Transconductance g_m in micromhos
		Average	Lowest	Highest			
6AG5	21	142	112	156	7	480	6,000
6AK5	30	46	36	58	10	460	5,500
6AS6*	4	46	42	52	10	450	5,800
6AU6	17	212	184	254	12	420	6,600
6BC5	6	130	120	142	8	590	5,800
6BH6*	3	224	—	—	10	—	—
6BC6*	10	174	144	194	12	410	7,300
6J4	2	200	—	—	15	321	12,000
6J6†	10	60	42	82	8	720	4,400
2C51†	8	40	38	46	8	550	5,400
404A	9	72	68	36	15	240	16,000

* Suppressor connected to plate and screen.
† Values are for a single section.

the presence of reflected electrons will affect the accuracy of induced grid noise predictions based on measured values of input admittance.

It is conceivable that the effect of reflected electrons could be eliminated or materially reduced by the use of a specially constructed tube. If this could be done, the remaining induced grid noise would be more completely correlated with shot noise. By properly detuning the input circuit of a suitable amplifier, it should then be possible to use this correlation to cause a partial cancellation of the effects of induced grid noise and thereby obtain substantially lower noise figures at high frequencies.

Acknowledgment

The authors would like to thank Professors W. G. Dow and G. Hok of the University of Michigan for helpful discussions during the course of this study.

On the Possibility of Amplification in Space-Charge-Potential-Depressed Electron Streams*

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Summary—**Hahn-Ramo theory is used to derive a characteristic wave equation for an electron stream whose single-valued velocity is a function of spatial co-ordinates. A means of solving this equation is found, for the particular case of two-dimensional Cartesian co-ordinates. A specific, but practical, linear velocity distribution is assumed. It is shown that for several types of boundary conditions, the only waves which can be set up in such a beam are purely propagational and not growing, bearing out the result derived by an approximate method by G. Kent.**

Numerical analysis for a cylindrical beam with potential depression was performed by a digital computer. As before, the results showed absence of any growing waves.

In order to check early results of Haeff,[1] which appeared to show the possibility of gain in single beam devices, an experiment was set up whereby a movable pickup cavity measured the amplitude of space-charge waves at a number of points along a drift tube.

Outputs were compared at different drift lengths for pulsed and continuous operation. No evidence of growing waves was observed, verifying the analytical results. It was found, however, that operation of the collector electrode at very low potentials created secondary electrons which returned to the gun region, were reflected, and then they flowed back with the primary beam. This double-stream action produced electronic gains up to 30 db. It is believed that either a similar effect, or else a space-charge wave gain produced as a direct result of nonuniform beam flow, can explain any signal gains found in conventional single-stream tubes.

Introduction

SEVERAL years ago, the double-stream amplifier, a mechanism for obtaining amplification in electron streams, was described by several authors.[1–4] It has been shown analytically and experimentally that the mixture of two homogeneous electron streams of slightly different velocity should give rise to exponentially increasing space-charge waves, the result of the perturbation of the two original sets of space-charge waves by one another. Other authors[5–8] have expanded this theory to general multiple-stream amplifiers, with finite boundaries. These theories indicate an optimum gain when only two distinct velocities are present. For the case of homogeneous mixtures of beams of different velocity, it has been shown that velocities in a Gaussian distribution cannot give rise to amplification.[8] A general proof given by Walker[9] indicates that a homogeneous mixture of electrons whose velocity distribution is flat or monotonic, or has a single maximum, cannot produce gain.

Some of the experimental data presented in Haeff's paper[1] indicated large amplification in the case of an electron stream coming from a single cathode. It was proposed that the observed gain was a consequence of the spatial velocity distribution caused by space-charge depression of potential. This had not been taken into account in the existing analyses, which dealt with a one-dimensional problem. A considerably more complicated problem results when the electron velocity is made a spatial function. Kent[10] has used a series approximation to show that gain cannot be self-consistent in a ribbon beam having space-charge depression of potential. The

* Original manuscript received by the IRE, September 24, 1954; revised manuscript received, January 7, 1955. Adapted from a dissertation submitted in partial fulfillment of the requirements for the Doctor of Philosophy Degree at the Univ. of Maryland, College Park, Md.

† RCA Laboratories, Princeton, N.J.

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